Lines and Circles

1. A line has the equation $x^2 + y^2 = 25$.
   What is the length of this line?

2. A circle has the equation $(x + 7)^2 + (y - 3)^2 = 25$.
   a What are the co-ordinates of the centre of this circle?
   b What is the area of the circle?

3. A circle has the equation $x^2 + y^2 = 49$.
   Another line has the equation $y = x + 2$.
   At what points do these two lines intersect each other?

4. A circle A has the equation $x^2 + y^2 = 81$.
   Another line B runs through the points (-12,-2) and (-10,1).
   a Find the equation of line B.
   b. Find the co-ordinates of the points where line B intersects circle A.

5. A circle A has the equation $(x - 5)^2 + \ (y - 8)^2 = 36$.
   Another line B runs through the points (-14,-2) and (-4,2).
   Find the equation where the line B intersects with circle A.

Success Criteria	**Completed**
I can identify the radius of a circle from the equation of the circle in the form $x^2 + y^2 = r^2$	
I can calculate the circumference of a circle using the formula $C = 2\pi r$	
I can calculate the area of a circle using the formula $A = \pi r^2$	
I can find the centre of a circle from the equation of the form: $(x - a)^2 + (x - b)^2 = r^2$	
I can find the gradient of a straight line given two points.	
I can find the intercept of a straight line given a gradient and a point.	
I can fine the equation of a straight line in the form $y = mx + c$	
I can solve simultaneous equations including an equation in quadratic form by substitution.	

Lines and Circles Examples

1 A line has the equation $x^2+y^2=64$.

What is the radius?

General equation for a circle is $x^2+y^2=r^2$ where r is the radius.

So the radius of $x^2+y^2=64$ is $\sqrt{64}=8$.

What is the circumference?

$$
\begin{aligned}
C &= 2\pi r \\
&= 2\times\pi\times 8 \\
&= 16\pi \\
&\simeq 50\cdot 2654826 \\
&\simeq 50\cdot 27 \text{ (to 2 d.p.)}
\end{aligned}
$$

What is the area?

$$
\begin{aligned}
A &= \pi r^2 \\
&= \pi\times 8\times 8 \\
&= 64\pi \\
&\simeq 201\cdot 0619298 \\
&\simeq 201\cdot 06 \text{ (to 2 d.p.)}
\end{aligned}
$$

2 $(x+a)^2+(y+b)^2=64$ would yield a circle of the same size as $x^2+y^2=64$.

The difference would be the co-ordinates of the centre of the circle. These would be $(-a,-b)$.

So a circle with the equation $(x-5)^2+(y+3)^2=64$ would have a radius of 8 and a centre at $(5,-3)$ as a minus x a minus is a plus and a plus times a minus is a minus.

3. A circle has the equation $x^2 + y^2 = 64$.
A line has the equation $y = x + 3$.
How do you find the intersecting points?

$$x^2 + y^2 = 64$$

Substitute $x+3$ for y.

$$\therefore \quad x^2 + (x+3)^2 = 64$$

	x	$+3$
x	x^2	$+3x$
$+3$	$+3x$	$+9$

$$\therefore \quad x^2 + x^2 + 6x + 9 = 64$$

$$\therefore \quad 2x^2 + 6x - 55 = 0$$

Use the Quadratic Formulae:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{for } ax^2 + bx + c = 0.$$

So, in $2x^2 + 6x - 55 = 0$, $a = 2$, $b = 6$ and $c = -55$.

$$x = \frac{-6 \pm \sqrt{36 + 440}}{4}$$

$$= \frac{-6 \pm \sqrt{476}}{4}$$

So $x = \frac{-6 + \sqrt{474}}{4}$ or $x = \frac{-6 - \sqrt{474}}{4}$

$= 3.942885264$ $\quad$ $= -6.942885264$

So, with these x-co-ordinates, we can substitute each one in turn into the equation to find the y-co-ordinates.

$y = x + 3$
$= 3.942885264 + 3$
$= 6.942885264$

$y = x + 3$
$= -6.942885264 + 3$
$= -3.942885264$

So $(x, y) = (3.942885264, 6.942885264)$ and $(-6.942885264, -3.942885264)$

4 Find the equation of a straight line from two points. ③

A line runs through the points $(4, 2)$ and $(-3, 5)$. What is the equation of the line?

Determine the left most point by comparing the x co-ords.

-3 is lower than 4 or $-3 < 4$.

$\text{Point}_1 = (-3, 5)$, $\text{Point}_2 = (4, 2)$

$x_1 = -3 \qquad y_1 = 5$

$x_2 = 4 \qquad y_2 = 2$

Find the gradient, m:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-5}{4-(-3)} = \frac{-3}{7}$$

Find the intercept:

At point $(4, 2)$, $x = 4$ and $y = 2$.

The equation for a straight line is $y = mx + c$

$$\therefore \quad 2 = -\frac{3}{7}(4) + c$$

$$\therefore \quad c = 2 + \frac{3}{7}(4)$$

$$= 2 + \frac{12}{7}$$

$$= \frac{14}{7} + \frac{12}{7}$$

$$= \frac{26}{7}$$

$$= 3\frac{5}{7}$$

So the equation of the line is $y = \frac{-3}{7}x + \frac{26}{7}$